Nuggets: Solubility: $K_{sp}$ Expressions, Molar Solubility, Solubility, Which Solid is More Soluble?, Reaction Quotient $Q$ (does a ppt form), pH Effects on Solubility, Common Ion Effect, Le Chatelier’s Principle

SOLUBILITY PRODUCT CONSTANT EXPRESSION

The reaction is always written: solid $\rightleftharpoons$ ions; the solid is not included in the equilibrium expression.

e.g., AgCl(s) $\rightleftharpoons$ Ag$^+$ (aq) + Cl$^-$ (aq); solubility product constant expression: $K_{sp} = [Ag^+][Cl^-]$.

$K_{sp}$ is the solubility product constant.

TYPICAL PROBLEMS:

- **Find molar solubility** from $K_{sp}$: write reaction, $K_{sp}$ expression, and ICE table; “x” in ICE Table is molar solubility (in M); plug into $K_{sp}$ expression and solve for x.

  **Solubility** (as opposed to molar solubility) usually has units of g/L (use molar mass to convert: (g/L) x (mol/g)).

  **Example 1**: a. What is the molar solubility of PbCl$_2$(s) given the $K_{sp} = 1.7 \times 10^{-5}$?  
  b. What is the solubility in g/L?  
  c. What are the concentration of Pb$^{2+}$ and Cl$^-$ after the solid has dissolved?

  **Answer 1: Part a. Step 1**: Write reaction; ICE Table

  PbCl$_2$(s) $\rightleftharpoons$ Pb$^{2+}$(aq) + 2Cl$^-$ (aq)

  I --- 0 0  
  C $-x$ $+x$ $+2x$  
  E --- $x$ 2x  

  **Step 2**: Write $K_{sp}$ expression

  $K_{sp} = [Pb^{2+}][Cl^-]^2$

  **Step 3**: Solve for x

  $1.7 \times 10^{-5} = (x)(2x)^2 = 4x^3$  
  $x = \text{molar solubility} = 1.6 \times 10^{-2} \text{ M}$

  **Part b. Convert M to g/L**

  $1.6 \times 10^{-2} \text{ M} = \frac{1.6 \times 10^{-2} \text{ mol PbCl}_2}{\text{L}}$

  $\frac{1.6 \times 10^{-2} \text{ mol PbCl}_2}{\text{L}} \left( \frac{278.1 g \text{ PbCl}_2}{1 \text{ mol PbCl}_2} \right) = 4.4 g \text{ PbCl}_2 / \text{L}$

  **Part c. Find [ion] from x**

  [Pb$^{2+}$] = $x = 1.6 \times 10^{-2}$M;  
  [Cl$^-$] = $2x = 2(1.6 \times 10^{-2}) = 3.2 \times 10^{-2}$M

- **Find $K_{sp}$ from molar solubility**: write reaction, $K_{sp}$ expression, and ICE table; “x” in ICE table is given in the problem; plug into $K_{sp}$ expression and solve for $K_{sp}$; if solubility given (g/L) convert $\rightarrow$ M (g/L x mol/g).

  **Example 2**: What is the $K_{sp}$ for Ca$_3$(PO$_4$)$_2$(s) if its molar solubility is $7.1 \times 10^{-7}$?

  **Answer 2: Step 1**: Write reaction and ICE Table

  Ca$_3$(PO$_4$)$_2$(s) $\rightleftharpoons$ 3Ca$^{2+}$(aq) + 2PO$_4^{3-}$(aq)

  I --- 0 0  
  C $-x$ $+3x$ $+2x$  
  E --- $3x$ 2x  

  **Step 2**: Write $K_{sp}$ expression

  $K_{sp} = [Ca^{2+}]^3[PO_4^{3-}]^2$

  **Step 3**: Substitute x into K expression

  $K_{sp} = (3x)^3(2x)^2 = (27x^3)(4x^2) = 108x^5$

  $x = \text{molar solubility} = 7.1 \times 10^{-7}$ M

  $K_{sp} = 108(7.1 \times 10^{-7})^5 = 1.9 \times 10^{-29}$
• Which solid is more soluble

- If same $K_{sp}$ expressions (e.g., $4x^3$), larger $K_{sp}$ will represent chemical that is more soluble
- If different $K_{sp}$ expressions (e.g., $x^2$ versus $4x^3$), molar solubility ($x$) must be calculated for each solid from ICE table and larger $x$ represents chemical that is more soluble

Example 3: Which is most soluble: AgCl ($K_{sp} = 1.8 \times 10^{-10}$), AgBr ($K_{sp} = 5.0 \times 10^{-13}$), Ag$_3$PO$_4$ ($K_{sp} = 1.4 \times 10^{-16}$)?

Answer 3: Step 1: AgCl vs AgBr

Since AgCl and AgBr have the same $K_{sp}$ expressions ($K_{sp} = x^2$), the $K_{sp}$ values can be compared directly. $K_{sp}$ (AgCl) > $K_{sp}$ (AgBr) → AgCl is more soluble.

Step 3: AgCl: Write reaction, ICE Table, solve for $x$.

$$\text{AgCl(s)} \rightleftharpoons \text{Ag}^{+}(aq) + \text{Cl}^{-}(aq)$$

| I | 0 | 0 |
| C | $-x$ | $+x$ | $+x$ |
| E | $-x$ | $x$ | $x$ |

$K_{sp} = \left[\text{Ag}^{+}\right]\left[\text{Cl}^{-}\right] = x^2$

$1.8 \times 10^{-10} = x^2$ ; $x = \sqrt{1.8 \times 10^{-10}}$

$x = 1.3 \times 10^{-5} M =$ solubility of AgCl

Step 4: Ag$_3$PO$_4$: Write reaction, ICE Table, solve for $x$.

$$\text{Ag}_3\text{PO}_4(s) \rightleftharpoons 3\text{Ag}^{+}(aq) + \text{PO}_4^{3-}(aq)$$

| I | 0 | 0 |
| C | $-x$ | $+3x$ | $+x$ |
| E | $-3x$ | $x$ | $x$ |

$K_{sp} = [\text{Ag}^{+}]^3[\text{PO}_4^{3-}] = (3x)^3(x) = 27x^4$

$1.4 \times 10^{-16} = 27x^4$ ; $x = \sqrt[4]{\frac{1.4 \times 10^{-16}}{27}}$

$x = 4.8 \times 10^{-5} M =$ solubility of Ag$_3$PO$_4$

Ag$_3$PO$_4$ is most soluble because its molar solubility (“$x$”) is largest.

• Does a precipitation form? use Reaction Quotient, $Q$

- $Q > K$ (over saturated; ppt forms); $Q < K$ (unsaturated; no ppt); $Q = K$ (equilibrium; saturated; no ppt)

Example 4: a. Does a precipitate form when 125ml 1.4 x 10$^{-3}$M solution of Pb(NO$_3$)$_2$ is mixed with 225ml 2.2 x 10$^{-5}$M NaF solution?

$K_{sp}$ PbF$_2$ = 3.7 x 10$^{-8}$

b. Given the [Pb$^{2+}$] concentration after the two solutions are mixed, what [F$^{-}$] is required for a precipitate to just form?

Answer 4: Part A. Step 1: Determine concentration of [Pb$^{2+}$] and [F$^{-}$] after mixing of solutions

For [Pb$^{2+}$]: $M_1 V_1 = M_2 V_2$: $(1.4 \times 10^{-3})(125) = M_2(350)$; $M_2 = [\text{Pb}^{2+}] = 5.0 \times 10^{-4} M$;

for [F$^{-}$]: $M_1 V_1 = M_2 V_2$: $(2.2 \times 10^{-5})(225) = M_2(350)$; $M_2 = [\text{F}^{-}] = 1.4 \times 10^{-5} M$;

Step 2: Write reaction and Q: PbF$_2(s) \rightleftharpoons \text{Pb}^{2+}(aq) + 2\text{F}^{-}(aq)$; $Q = [\text{Pb}^{2+}][\text{F}^{-}]^2$;

Step 3: Calculate Q: $Q = (5.0 \times 10^{-4})(1.4 \times 10^{-5})^2 = 9.8 \times 10^{-14}$;

Step 4: Compare Q to $K_{sp}$; $9.8 \times 10^{-14} < 3.7 \times 10^{-8} \rightarrow Q < K_{sp} \rightarrow$ no precipitate forms!

Part B. Step 1: Precipitate occurs when $Q > K_{sp}$; solve for [F$^{-}$] when $Q = K_{sp}$: $3.7 \times 10^{-8} = [\text{Pb}^{2+}][\text{F}^{-}]^2$; $3.7 \times 10^{-8} = (5.0 \times 10^{-4})[\text{F}^{-}]^2$;

$[\text{F}^{-}]^2 = 7.4 \times 10^{-5}$; [F$^{-}$] = 8.6 x 10$^{-5}$M;

Step 2: when $[\text{F}^{-}] > 8.6 \times 10^{-3} \rightarrow Q > K_{sp} \rightarrow$ a precipitate forms!

• How does the pH affect solubility

- if the ions form a weak acid then adding an acid will increase the solubility;
- if the ions form a strong acid then pH has no effect on solubility

Example 5: Which solid, AgF(s), AgCl(s), AgBr(s), or AgI(s), is more soluble in an acidic solution (as compared to pure water)?

Answer 5: AgF

Write reaction of H$^+$ reacting with AgCl: AgCl(s) + H$^+$ (aq) $\rightleftharpoons$ Ag$^+$ (aq) + HCl(aq); since HCl cannot form in water since it is a strong acid (strong acids dissociate 100% in water), this reaction does not proceed; same results for AgBr(s) and AgI(s) since HBr and HI are strong acids too;

Now with AgF: AgF(s) + H$^+$ (aq) $\rightleftharpoons$ Ag$^+$ (aq) + HF(aq); since HF is a weak acid, it can form in water and reaction can proceed;

AgF dissolves more in acidic solution than in pure water

Another way to think of this is using Le Chatelier’s Principle: In the reaction AgF(s) $\rightleftharpoons$ Ag$^+$ (aq) + F$^-$ (aq) when H$^+$ is added it reacts with F$^-$ and forms HF; this effectively reduces [F$^-$] and from Le Chatelier’s Principle, if [F$^-$] $\downarrow$ reaction shifts right, as reaction shifts right the solid, AgF(s), dissolves more and the solubility increases
pH from $K_{sp}$

**Example 6:** What is the pH when Pb(OH)$_2$(s) dissolves in water? $K_{sp} = 1.2 \times 10^{-15}$?

<table>
<thead>
<tr>
<th>Answer 6: Step 1: Write reaction and ICE Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pb(OH)$_2$(s) $\rightleftharpoons$ Pb$^{2+}$(aq) + 2OH$^-$ (aq)</td>
</tr>
<tr>
<td>Step 2: Write K expression</td>
</tr>
<tr>
<td>$K_{sp} = [\text{Pb}^{2+}][\text{Cl}^-]^2$</td>
</tr>
<tr>
<td>Step 3: Plug in values and solve for x:</td>
</tr>
<tr>
<td>$1.2 \times 10^{-15} = (x)(2x)^2 = 4x^3$; $x = 6.69 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

From ICE Table: $[\text{OH}^-] = 2x = (6.69 \times 10^{-6}) = 1.34 \times 10^{-5}$; $\text{pOH} = -\log([\text{OH}^-]) = -\log(1.34 \times 10^{-5}) = 4.87$; pH = 14.00 - 4.87 = 9.13

**Common Ion Effect:** Adding a *common ion* from 2 sources: The ion from the solid dissolving and the ion already in the solution; this changes the initial starting conditions and from Le Chatelier's principle, shifts the reaction so solid is less soluble; same calculation but ICE Table Initial row is different

**Example 7:** What is the molar solubility of PbCl$_2$(s) in a 0.4M NaCl(aq) solution given the $K_{sp} = 1.7 \times 10^{-5}$?

<table>
<thead>
<tr>
<th>Answer 7: Step 1: Write reaction and ICE Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>PbCl$_2$(s) $\rightleftharpoons$ Pb$^{2+}$(aq) + 2Cl$^-$ (aq)</td>
</tr>
<tr>
<td>Step 2: Write K expression</td>
</tr>
<tr>
<td>$K_{sp} = [\text{Pb}^{2+}][\text{Cl}^-]^2$</td>
</tr>
<tr>
<td>Step 3: Plug in values and solve for x:</td>
</tr>
</tbody>
</table>
| $1.7 \times 10^{-5} = (x)(0.4)^2 = 4x^3$; check to see if approximation, $0.4+2x = 0.4$ can be made: (100)$K_{sp}$ < [ ]; $100(1.7 \times 10^{-5}) < 0.4M$ true $\rightarrow$ valid to make approximation; $1.7 \times 10^{-5} = (x)(0.4)^2$; $x = 1.1 \times 10^{-4}$M; In pure water as shown in the first example, solubility was 1.6 $\times 10^{-2}$; it has now dropped by a factor of $\sim$100

**Factors Affecting Solubility** (*the first 2 are usually covered in more detail*)

1. **Acidity** (pH) – adding an acid increases solubility (when a weak acid forms) as compared to dissolving in water or has no effect on solubility (when a strong acid forms)

2. **Common ions** in solution (common ion effect) – dissolving MX in X$^-$ solution decreases solubility; for example, dissolving AgCl in a 0.1M Cl$^-$ solution as compared to water: solubility decreases

3. Complex ion formation – another chemical binds with one of the components in MX increasing solubility (e.g., AgCl(s) dissolving in an ammonia, NH$_3$, solution forms Ag(NH$_3$)$_2^+$)

1. a. What is the molar solubility (M) of AgCl(s)? ($K_{sp} = 1.8 \times 10^{-10}$)?
   b. What is the solubility (in g/L) of AgCl(s)?

2. What is the molar solubility (M) of Cu$_2$S(s)? ($K_{sp} = 1.6 \times 10^{-48}$)?

3. a. Calculate the $K_{sp}$ for FeS(s) if its solubility is 2.21 $\times 10^{-9}$ M.
   b. Calculate the $K_{sp}$ for Ag$_2$SO$_4$(s) if its solubility is 1.44 $\times 10^{-2}$ M.

4. If 0.050g SrF$_2$(s) is placed in 250.ml water, how many grams will dissolve? don’t dissolve? ($K_{sp} = 4.3 \times 10^{-9}$)

5. If a maximum of 10.9g SnI$_2$(s) will dissolve in 1.00L of water, what is the $K_{sp}$ of SnI$_2$(s)?

6. I. Which is most soluble?
   - PbF$_2$(s) ($K_{sp} = 3.7 \times 10^{-8}$), PbCl$_2$(s) ($K_{sp} = 1.7 \times 10^{-5}$), PbBr$_2$(s) ($K_{sp} = 6.3 \times 10^{-6}$), PbI$_2$(s) ($K_{sp} = 8.7 \times 10^{-9}$)
   - II. Which is most soluble?
     - AgI(s) ($K_{sp} = 1.5 \times 10^{-16}$), Ag$_3$PO$_4$(s) ($K_{sp} = 1.3 \times 10^{-20}$), or Ag$_2$S(s) ($K_{sp} = 1 \times 10^{-49}$)
7. Which compound will **not** dissolve more in an acidic solution?
   a. PbF\(_2\)(s)  
   b. Pb(CN)\(_2\)(s)  
   c. PbI\(_2\)(s)  
   d. Pb\(_3\)(PO\(_4\))\(_2\)(s)

8. Ag\(_2\)CrO\(_4\)(s) will be most soluble in which solution? (K\(_{sp}\) = 2.4 x 10\(^{-12}\))
   a. 0.10 M K\(_2\)CrO\(_4\)(aq)  
   b. 0.40 M AgNO\(_3\)(aq)  
   c. 4.0 M AgNO\(_3\)(aq)  
   d. pure water

9. What is the molar solubility of MgF\(_2\)(s) in 0.050M NaF(aq)? (K\(_{sp}\) = 5.2 x 10\(^{-11}\))

10. a. Zn(CN)\(_2\)(s) is more soluble when a strong acid is added to an aqueous solution. Write the reaction when H\(^+\)(aq) is added to Zn(CN)\(_2\)(s).
    b. What is K\(_{eq}\) for the reaction written in part “a” given K\(_{sp}\) Zn(CN)\(_2\)(s) = 8.0 x 10\(^{-12}\) and K\(_a\) HCN = 4.0 x 10\(^{-10}\)?

11. a. ScF\(_3\)(s) is more soluble when a strong acid is added to an aqueous solution. Write the reaction when H\(^+\)(aq) is added to ScF\(_3\)(s).
    b. What is K\(_{eq}\) for the reaction written in part “a” given K\(_{sp}\) ScF\(_3\)(s) = 5.8 x 10\(^{-24}\) and K\(_a\) HF = 7.2 x 10\(^{-4}\)?

12. The solubility of M\(_2\)S(s) is 2.0 x 10\(^{-5}\)M. If solid M\(_2\)S(s) is added to water which statement is correct?
   a. [M\(^+\)] = 2.0 x 10\(^{-5}\)M, [S\(^-2\)] = 2.0 x 10\(^{-5}\)M
   b. [M\(^+\)] = 4.0 x 10\(^{-5}\)M, [S\(^-2\)] = 2.0 x 10\(^{-5}\)M
   c. [M\(^+\)] = 2.0 x 10\(^{-5}\)M, [S\(^-2\)] = 4.0 x 10\(^{-5}\)M
   d. [M\(^+\)] = 2.0 x 10\(^{-5}\)M, [S\(^-2\)] = 1.0 x 10\(^{-5}\)M
   e. [M\(^+\)] = 1.0 x 10\(^{-5}\)M, [S\(^-2\)] = 2.0 x 10\(^{-5}\)M

13. What concentration of NaF(aq) is required to just cause a 5.0 x 10\(^{-5}\)M Mg(NO\(_3\))\(_2\)(aq) solution to precipitate? (K\(_{sp}\) MgF\(_2\)(s) = 5.2 x 10\(^{-11}\))

14. If 0.100g Na\(_2\)CO\(_3\)(s) is added to a 200.ml solution of 1.00 x 10\(^{-3}\)M Ca(NO\(_3\))\(_2\)(aq), will a precipitate form? (K\(_{sp}\) CaCO\(_3\)(s) = 3.8 x 10\(^{-9}\))

15. Will a precipitate of neodymium carbonate, Nd\(_2\)(CO\(_3\))\(_3\)(s), form if 250ml 1.0 x 10\(^{-5}\) M Nd(NO\(_3\))\(_3\)(aq) is mixed with 150ml 5.5 x 10\(^{-3}\) M Na\(_2\)CO\(_3\)(aq)? (K\(_{sp}\) Nd\(_2\)(CO\(_3\))\(_3\) = 1.0 x 10\(^{-33}\))
   a. Yes because Q < K\(_{sp}\)  
   b. No because Q < K\(_{sp}\)  
   c. Yes because Q > K\(_{sp}\)  
   d. No because Q < K\(_{sp}\)  
   e. No because Q = K\(_{sp}\)

16. If 125.0ml 0.095M Na\(_3\)PO\(_4\)(aq) is combined with 250.0ml 0.0055M AgNO\(_3\)(aq), will a precipitate form? (K\(_{sp}\) Ag\(_3\)(PO\(_4\))(s) = 8.9 x 10\(^{-17}\))

17. The K\(_{sp}\) for Ni(OH)\(_2\)(s) is 5.5 x 10\(^{-16}\). What is the pH of a saturated solution of Ni(OH)\(_2\)(aq)?

18. If a solution contains 3.5 x 10\(^{-1}\) M Ag\(^+\)(aq) and 4.6 x 10\(^{-4}\) M Ba\(^{2+}\)(aq), and this solution is titrated with Na\(_2\)SO\(_4\)(aq), which solid would precipitate first, Ag\(_2\)SO\(_4\)(s) or K\(_{sp}\) BaSO\(_4\)(s)?
   (K\(_{sp}\) Ag\(_2\)SO\(_4\)(s) = 1.2 x 10\(^{-5}\) and K\(_{sp}\) BaSO\(_4\)(s) = 1.5 x 10\(^{-9}\))
ANSWERS

1. a. $1.3 \times 10^{-5}$ M \[\text{AgCl(s) } \rightleftharpoons \text{Ag}^+(aq) + \text{Cl}^-(aq); \text{ write ICE Table:} \]

\[
K_{sp} = [\text{Ag}^+] [\text{Cl}^-]; 1.8 \times 10^{-10} = x^2; x = 1.34 \times 10^{-5} \text{M} = \text{molar solubility} \]

b. \[1.9 \times 10^{-3} \text{ g/L} \{1.34 \times 10^{-2} \text{mol AgCl/L} \times (143.49 \text{g AgCl}/1 \text{mol AgCl}) = 1.92 \times 10^{-3} \text{g/L} \]

2. \[7.4 \times 10^{-17} \text{ M} \{\text{Cu}_2S(s) \rightleftharpoons 2\text{Cu}^+(aq) + S^2-(aq); \text{ write ICE Table:} \]

plug into $K_{sp} = [\text{Cu}^+]^2[\text{S}^-^2]; 1.6 \times 10^{-48} = (2x)^2x = 4x^3; x = 7.37 \times 10^{-17} \text{M} \]

3. a. \[K_{sp} = 4.88 \times 10^{-18} \{\text{FeS(s) } \rightleftharpoons \text{Fe}^{2+}(aq) + \text{S}^2-(aq); \text{ write ICE Table:} \]

substitute into $K_{sp} = [\text{Fe}^{2+}][\text{S}^2^-] = x^2; x = \text{molar sol} = 2.21 \times 10^{-9}; \]

\[
K_{sp} = x^2 = (2.21 \times 10^{-9})^2 = 4.88 \times 10^{-18} \]

b. \[K_{sp} = 1.19 \times 10^{-5} \{\text{Ag}_2\text{SO}_4(s) \rightleftharpoons 2\text{Ag}^+(aq) + \text{SO}_4^{2-}(aq); \text{ write ICE Table:} \]

plug into $K_{sp} = [\text{Ag}^+]^2[\text{SO}_4^{2-}] = (2x)^2x = 4x^3; x = \text{molar sol} = 1.44 \times 10^{-2}; \]

\[
K_{sp} = 4x^3 = 4(1.44 \times 10^{-2})^3 = 1.19 \times 10^{-5} \]

4. 0.018g don’t dissolve; 0.0320g dissolve \[\{\text{SrF}_2(s) \rightleftharpoons \text{Sr}^{2+}(aq) + 2\text{F}^-(aq); \text{ write ICE Table:} \]

\[
K_{sp} = [\text{Sr}^{2+}][\text{F}^-]^2; K_{sp} = (x)(2x)^2 = 4x^3 = 4.3 \times 10^{-9}; \]

\[
x = 1.02 \times 10^{-3} \text{M} = \text{solubility of SrF}_2; \]

mol SrF\text{\textsubscript{2}} = M x L = (1.02 \times 10^{-3})(0.250) = 0.000255mol Sr\text{\textsubscript{F}}\text{\textsubscript{2}}; \]

0.000255mol Sr\text{\textsubscript{F}}\text{\textsubscript{2}} x (125.3 g Sr\text{\textsubscript{F}}\text{\textsubscript{2}}/1 mol Sr\text{\textsubscript{F}}\text{\textsubscript{2}}) = 0.0320g dissolves in 250ml water \]

5. \[K_{sp} = 9.96 \times 10^{-5} \{\text{SnI}_2(s) \rightleftharpoons \text{Sn}^{2+}(aq) + 2\text{I}^-(aq); \text{ write ICE Table:} \]

\[
K_{sp} = [\text{Sn}^{2+}][\text{I}^-]^2; K_{sp} = (x)(2x)^2 = 4x^3 = 9.96 \times 10^{-5} \]

6. I. PbCl\text{\textsubscript{2}} \{since all of the chemicals have the same $K_{sp}$ expressions, $4x^3$, their $K_{sp}$ values can be compared directly with the largest $K_{sp}$ \rightarrow \text{most soluble}; PbCl\text{\textsubscript{2}} has the largest value and is most soluble} \]

II. Ag\text{\textsubscript{3}}\text{PO}_4 \{since all of the chemicals have different $K_{sp}$ expressions each molar solubility (“x”) need to be calculated and compared; the largest “x” \rightarrow \text{most soluble}; \]

\[
\text{AgI(s) } \rightleftharpoons \text{Ag}^+(aq) + \Gamma^-(aq); \text{ write ICE Table for AgI}; K_{sp} = [\text{Ag}^+][\Gamma]; \]

\[
K_{sp} = (x)(x) = x^2 = 1.5 \times 10^{-16}; x = 1.22 \times 10^{-8}; \]

\[
\text{Ag}_3\text{PO}_4(s) \rightleftharpoons 3\text{Ag}^+(aq) + \text{PO}_4^{3-}(aq); \text{ write ICE Table for Ag}_3\text{PO}_4; \]

\[
K_{sp} = [\text{Ag}^+]^3[\text{PO}_4^{3-}]; K_{sp} = (3x)^3(x) = 27x^4 = 1.3 \times 10^{-20}; x = 4.68 \times 10^{-6}; \]

\[
\text{Ag}_2\text{S(s) } \rightleftharpoons 2\text{Ag}^+(aq) + \text{S}^2-(aq); \text{ write ICE Table for Ag}_2\text{S}; K_{sp} = [\text{Ag}^+]^2[\text{S}^2^-]; \]

\[
K_{sp} = (2x)^2(x) = 4x^3 = 1 \times 10^{-49}; x = 2.9 \times 10^{-17}; \]

Ag\text{\textsubscript{3}}\text{PO}_4 has largest x value = most soluble} \]
7. c  look at the anions in each compound; for PbF₂, it is F⁻; let the anions react with H⁺; H⁺(aq) + F⁻(aq) → HF(aq), if the acid formed is a weak acid, then it can form (i.e., HF will form); the reaction PbF₂(s) → Pb⁺²(aq) + 2F⁻(aq) will have the [F⁻] as the H⁺ is add and from Le Chatelier’s Principle will pull the reaction to the right and more solid PbF₂(s) will dissolve; this will be true with other anions forming weak acids: HCN and H₃PO₄; HI will not form since it is a strong acid (strong acids don’t form in water since they dissociate 100%) so adding H⁺(aq) will not shift the reaction, Pbl₂(s) → Pbl⁺²(aq) + 2I⁻(aq), and its solubility will be independent of the acidity.

8. d  from the reaction, Ag₂CrO₄(s) → 2Ag⁺(aq) + CrO₄²⁻(aq), which ever conditions shifts the reaction most to the right is the solution in which Ag₂CrO₄(s) will be most soluble; “a”, “b”, and “c” add a common ion and shift the reaction to the left (less soluble); pure water will therefore be the solution with the highest solubility.

9. 2.1 x 10⁻⁸ M  {MgF₂(s) → Mg⁺²(aq) + 2F⁻(aq); write ICE Table:

\[
K_{sp} = [Mg⁺²][F⁻]²; 5.2 x 10⁻¹¹ = x(0.05+2x)²; \text{since (100)5.2 x 10⁻¹¹ < [F⁻]₀ make approximation: 5.2 x 10⁻¹¹ = x(0.05)²}; x = 2.08 x 10⁻⁶ M = molar solubility
\]

10. a. Zn(CN)₂(s) + 2H⁺(aq) → Zn⁺²(aq) + 2HCN(aq)

b.  \[K_{eq} = 5.0 x 10⁻⁷\]  {The reaction in part “a” can be written as the sum of 2 reactions:

I. Zn(CN)₂(s) → Zn⁺²(aq) + 2CN⁻(aq)  and
II. 2H⁺(aq) + 2CN⁻(aq) → 2HCN(aq)

K for Reaction I = \(K_{sp}\); K for Reaction II = \(\frac{1}{K_{a}}\); \[K_{eq} = (K_{sp})\frac{1}{K_{a}} = (8.0 x 10⁻¹²)(1/4.0 x 10⁻¹⁰)² = 5.0 x 10⁻⁷\]

11. a. ScF₃(s) + 3H⁺(aq) → Sc⁺³(aq) + 3HF(aq)

b.  \[K_{eq} = 1.55 x 10⁻¹⁴\]  {The reaction in part “a” can be written as the sum of 2 reactions:

I. ScF₃(s) → Sc⁺³ + 3F⁻  and
II. 3H⁺(aq) + 3F⁻(aq) → 3HF(aq)

K for Reaction I = \(K_{sp}\); K for Reaction II = \(\frac{1}{K_{a}}\); \[K_{eq} = (K_{sp})\frac{1}{K_{a}} = (5.8 x 10⁻²⁴)(1/7.2 x 10⁻⁴)³ = 1.55 x 10⁻¹⁴\]

12. b  {M₂S has solubility of 2.0 x 10⁻⁵ M and there are 2 M⁺ for each M₂S so [M] = 4.0 x 10⁻⁵ M and there is 1 S⁻² for each M₂S so [S⁻²] = 2.0 x 10⁻⁵ M; can also set up an ICE Table as shown}

\[
M₂S(s) \rightarrow 2M⁺(aq) + S⁻²(aq) \quad (\text{ICE Table})
\]

\[
\begin{array}{ccc}
\text{M₂S(s)} & \rightarrow & 2\text{M⁺(aq)} + \text{S⁻²(aq)} \\
\text{I} & \rightarrow & 0 \\
\text{C} & \rightarrow & -2.0 \times 10⁻⁵ \\
\text{E} & \rightarrow & 4.0 \times 10⁻⁵ \\
\end{array}
\]

\[
K_{sp} = [M⁺²][S⁻²] = 5.2 x 10⁻¹¹ (5.0 x 10⁻⁵)² = 2.6 x 10⁻²⁶; \text{when } [F⁻] > 1.02 x 10⁻³ M \rightarrow Q > K_{sp} \rightarrow \text{precipitate}
\]

13. [F⁻] > 1.02 x 10⁻³ M  {MgF₂(s) → Mg⁺²(aq) + 2F⁻(aq); \[K_{sp} = [Mg⁺²][F⁻]²; 5.2 x 10⁻¹¹ = (5.0 x 10⁻⁵)²; [F⁻] = 1.02 x 10⁻³ M; when } [F⁻] > 1.02 x 10⁻³ M \rightarrow Q > K_{sp} \rightarrow \text{precipitate}

14. Precipitate forms  {CaCO₃(s) → Ca⁺²(aq) + CO₃²⁻(aq); [CO₃²⁻] = 4.72 x 10⁻³ M; [Ca⁺²] = 1 x 10⁻³ M;
Q = [Ca⁺²][CO₃²⁻] = (1 x 10⁻³)(4.72 x 10⁻³) = 4.72 x 10⁻⁶; since Q > K_{sp} → a precipitate forms}

15. c  {solutions are mixed so use \(M_1V_1 = M_2V_2\) to find new concentrations after mixing:
for Nd(NO₃)₃: (1.0 x 10⁻⁵)(250) = M₂(400); \[Nd(NO₃)₃ = 6.25 x 10⁻⁶ M; \]
for Na₂CO₃: (5.5 x 10⁻³)(150) = M₂(400); \[Na₂CO₃ = 2.06 x 10⁻³ M; \]
reaction: \(Nd₂(CO₃)₃(s) \rightarrow 2Nd⁺³(aq) + 3CO₃²⁻(aq); Q = [Nd⁺³]²[CO₃²⁻]³ = (6.25 x 10⁻⁶)²(2.06 x 10⁻³)³ = 3.4 x 10⁻¹⁹; \}
since Q > K_{sp} there will be a precipitate}
16. Precipitate forms \( \text{Ag}_3\text{PO}_4(s) \rightleftharpoons 3\text{Ag}^+(aq) + \text{PO}_4^{3-}(aq) \); solutions are mixed so use \( M_1V_1 = M_2V_2 \) to find new concentrations after mixing:

for \( [\text{PO}_4^{3-}] \): \((0.095)(125) = M_2(375); [\text{PO}_4^{3-}] = 0.0317 \text{M} \);

for \( [\text{Ag}^+] \): \((0.0055)(250) = M_2(375); [\text{Ag}^+] = 0.00367 \text{M} \);

\[ Q = [\text{Ag}^+]^3[\text{PO}_4^{3-}] = (0.00367)^3(0.0317) = 1.57 \times 10^{-9}; \text{since } Q > K_{sp} \rightarrow \text{precipitate forms} \]

17. pH = 9.01 \( \text{Ni(OH)}_2(s) \rightleftharpoons \text{Ni}^{2+}(aq) + 2\text{OH}^-(aq) \); write ICE Table;

\[ K_{sp} = [\text{Ni}^{2+}][\text{OH}^-]^2; K_{sp} = (x)(2x)^2; \text{find } \text{“x”}; 4x^3 = 5.5 \times 10^{-16}; x = 5.16 \times 10^{-6}; \]

\[ [\text{OH}^-] = 2x = 2(5.16 \times 10^{-6}) = 1.03 \times 10^{-5} \text{M}; \text{pOH} = 4.99; \text{pH} = 9.01 \]

18. BaSO\(_4\) (s) \( \{ \text{Find the maximum [SO}_4^{2-}] \text{just before precipitate occurs for each chemical:} \)

\( \text{Ag}_2\text{SO}_4(s) \rightleftharpoons 2\text{Ag}^+(aq) + \text{SO}_4^{2-}(aq); K_{sp} = [\text{Ag}^+]^2[\text{SO}_4^{2-}]; 1.2 \times 10^{-5} = [3.5 \times 10^{-1}]^2[\text{SO}_4^{2-}]; [\text{SO}_4^{2-}] = 9.80 \times 10^{-5} \text{M} \);

\( \text{BaSO}_4(s) \rightleftharpoons \text{Ba}^{2+}(aq) + \text{SO}_4^{2-}(aq); K_{sp} = [\text{Ba}^{2+}][\text{SO}_4^{2-}]; 1.5 \times 10^{-9} = [4.6 \times 10^{-4}][\text{SO}_4^{2-}]; [\text{SO}_4^{2-}] = 3.26 \times 10^{-6} \text{M} \);

since a smaller \([\text{SO}_4^{2-}]\) will precipitate \( \text{BaSO}_4 \rightarrow \text{BaSO}_4 \) will precipitate first}