

### Exam 3

**Calculators are not allowed for this exam.**

**You will find that for many questions it is useful to know that  $e^{-1} \approx 0.4$  and that other powers can be obtained using  $e^{-n} = (e^{-1})^n \approx (0.4)^n$ .**

1. This exam contains 9 pages of questions and instructions, three pages of equations, a page of wavefunctions, a periodic table, and a page of constants and conversion factors.
2. Show your work and make your reasoning clear.
3. You have 2.0 hours to work on the exam.

1. \_\_\_\_\_/20

2. \_\_\_\_\_/25

3. \_\_\_\_\_/15

4. \_\_\_\_\_/20

4. \_\_\_\_\_/20

Total \_\_\_\_\_/100

Answer the following 4 miscellaneous questions about entropy and collision theory.

1a. (5 pts). What is the statistical prediction for the entropy of a perfect crystal at  $T=0$  ? Give a mathematical explanation.

1b. (5 pts.) If there are two ways to orient each molecule in a crystal, how many ways  $W$  are there to orient a mole ( $N_A$  – Avagardo's number) of such molecules? Use that result to calculate the entropy of the crystal of  $N_A$  molecules.

1c (5 pts). Qualitatively explain how the entropy would be different for a gas of indistinguishable and non-interacting polyatomic versus monatomic molecules.

1d. (5 pts.) Consider an elementary bimolecular reaction. If the masses of the gaseous molecules magically increased their masses by a factor of three (such as by forming non-reactive trimers), what does collision theory predict for the change in the rate constant?

2. To estimate the population of a state, scientists often talk about energy levels in terms of  $kT$ . Answer the following questions to learn why.

2a. (5 pts.) Derive the equation for  $q^v$  for a harmonic oscillator.

2b. (5 pts) Write an expression that gives the population of the eigenstate  $\varepsilon_i$  for a harmonic oscillator.

2c. (10 pts) Consider a harmonic oscillator that has an energy level spacing of  $kT$ . For each eigenstate drawn below, label its population.

\_\_\_\_\_  $3kT$

\_\_\_\_\_  $2kT$

\_\_\_\_\_  $kT$

\_\_\_\_\_  $0$

2d. (5 pts) In lecture, we found that  $I_2$ , which has a frequency of  $\tilde{\nu} = 215 \text{ cm}^{-1}$ , has 23% of its population in the  $\nu=1$  level and that  $Cl_2$ , which has a frequency of  $\tilde{\nu} = 560 \text{ cm}^{-1}$ , has 6% of its population in the  $\nu=1$  level. Using your results above, what is the approximate population of  $\nu=1$  for  $BrI$ , which has a frequency of  $\tilde{\nu} = 410 \text{ cm}^{-1}$ ? Show your work.

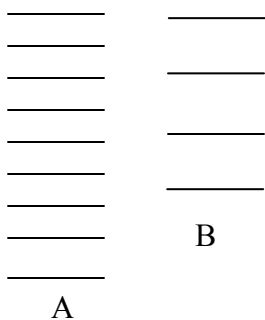
Answer the following 2-part question to learn about the density of the atmosphere.

3a. (5 pts.) Write the expression for the ratio of the number of molecules in a state with energy  $\epsilon_1$  to the number in a state with energy  $\epsilon_0$  at a temperature  $T$ .

3b. (10 pts.) The energy of a particle in a gravitational field is  $\epsilon = mgh$  where  $m$  is the mass of the particle,  $h$  is the height above a reference point, and  $g \approx 10 \text{ m/s}^2$  is the acceleration of gravity. Estimate the ratio of the number density of  $\text{O}_2$  at 8314 m compared to that at sea level. Assume that the temperature is 320 K.

4a. (5 pts.) Derive the equation for the equilibrium constant  $K$  for the chemical reaction  $A \leftrightarrow B$ , where the eigenstates of  $A$  and  $B$  are spaced by  $\epsilon_A$  and  $\epsilon_B$ , respectively, and the relative energy of  $B$  is offset by  $\Delta\epsilon_{AB}$ . Start from the definition of  $K = [B]/[A]$ .

4b. (5 pts) If the eigenstates have the relative spacings drawn below, explain whether the equilibrium constant will favor  $A$  or  $B$  and why.



4c. (5 pts) Calculate  $K$  for  $\epsilon_A = kT$ ,  $\epsilon_B = 2kT$  and  $\Delta\epsilon_{AB} = 2kT$  for vibrational mode in the high temperature limit. Round your answer to 1 significant digit.

4d. (5 pts) To make  $K = 1$ , what value would  $\Delta\epsilon_{AB}$  need to be? Write an expression (do not evaluate).

5a. (5 pts) Write out the equation for the canonical partition function  $Q$  in terms of the molecular translational, rotational, and vibrational partition functions (e.g.  $q^T$ ,  $q^R$ ,  $q^V$ ) for a homonuclear diatomic gas in the high temperature limit.

5b. (5 pts.) Find the equation for  $\langle E^T \rangle$  for the homonuclear diatomic gas.



5d. (10 pts) Shown below is  $C_p$  measured as a function of  $T$  for  $\text{Cl}_2$ ,  $\text{Br}_2$  and  $\text{I}_2$ . Answer the following questions, assuming that the gases are non-interacting. As a reminder,

$$C_p = C_v + R, \text{ and that } C_v = \left( \frac{\partial \langle E \rangle}{\partial T} \right)_v .$$

1. What values of  $C_p$  should  $\text{Cl}_2$ ,  $\text{Br}_2$  and  $\text{I}_2$  reach as the temperature is raised?
2. At  $T = 300 \text{ K}$ , why does  $\text{Cl}_2$  have a much lower value of  $C_p$  than  $\text{I}_2$ ?
3. What value would you expect  $C_p$  to reach at high  $T$  for a linear triatomic like  $\text{HCN}$ ? Explain.

