PRINT NEATLY *** USE A BLACK PEN *** DO NOT STAPLE

Course 565/665 Lecture Number 2 Date 1/23/03

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All future lectures will be held B383.

LAST TIME

$$P_A = \frac{n_A}{N}$$

Ex] Event A - Rolling a die and getting a 3.

$$P_A = \frac{n_A}{N} = \frac{1}{6}$$

Mutually Exclusive (ME) - ME precludes any other outcome.

Collectively Exhaustive (CE) - CE if they represent the entire set of possible outcomes.

Independent Events (IE) - IE are events that are not related to each other.

NOTE - Events cannot be ME and also independent.

Multiplicity - The total number of ways in which all different outcomes can occur. Defined as W.

Example - 3 cars, two colors.

Rolls Royce	White
Fiat	Black
Chevy	

n_A (R, F, C)

n_B (W, B)

$$W = n_A n_B$$

To determine multiplicity, one must multiply the number of possible events from each category (A, B, etc) together.

$W = 3 \cdot 2 = 6$ possibilities.

• Combination of Events

- Addition Rule - Outcomes A, B, ... are ME with probabilities $P_A, P_B, \ldots$

$$P_A = \frac{n_A}{N} \qquad P_B = \frac{n_B}{N}$$

$$P_A(\text{or } B \text{ or } C, \text{etc}) = \frac{n_A + n_B + n_C + \ldots}{N} = P_A + P_B + \ldots$$

These events must be OR and ME to apply the Addition Rule.

Head + Tail is the sum of all possibilities. These are CE.

$$N = n_A + n_B$$

$$P_A + P_B = \frac{n_A + n_B}{N} = \frac{N}{N} = 1$$

Multiplication Rule (Events must be AND and IE)

Outcomes are IE with probabilities $p_A, p_B, \ldots$

Probability of outcome A AND outcome B.

$$P_A P_B = \left(\frac{n_A}{N}\right)\left(\frac{n_B}{N}\right) = \frac{n_A n_B}{N}$$

Examples: Rolling a die. What is the probability of rolling a 2 OR a 5 in a single die rule.

- ME, OR, so we must use the Addition Rule.

$$P_A = \frac{1}{6} \qquad P_B = \frac{1}{6}$$

$$P_A + P_B = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

② Roll a die twice: We want a 2 on the 1st roll and a 5 on the second roll.

$$P = P_A P_B = \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) = \frac{1}{36}$$

We use the Multiplication Rule since the events are IE and there is an AND.

③ Probability of getting 5 heads on a coin. These events are independent and we are asking an AND question.

$$P = P_H P_H P_H P_H P_H = (P_H)^5 = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$$

This would be the same probability as getting HHHTH, HTTTT, etc.

A General Rule for tossing coins:

$$P_{(n_A,N)} = P_H^{n_A} P_T^{(N-n_A)}$$

$$= P_H^{n_A} P_H^{N-n_A} \quad \text{since } P_H = P_T = \frac{1}{2}$$

$$= P_H^{N+n_A-n_A} = P_H^N$$

More on Combination of events (independent events)

Given events A and B.

$P_{AB} = P_A P_B$

What is the probability that B does not occur?

$P_{B \text{ NOT occuring}} = [1 - P_B] = P_{\text{NOT } B}$

$P_{A, \text{NOT } B} = P_A [1 - P_B] = P_A - P_A P_B$

The above is the probability of getting A on the first trial and the probability of not getting B on the second (AND, IE).

- Neither A nor B

$P_{\text{NOT A, NOT B}} = (1 - P_A)(1 - P_B)$