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Course 565/665 Lecture Number ______ Date 1/27/03

Lecturer Dr. Silvia Cavagnero Note Taker Eric Fulmer

From Last Time (See previous lecture notes for a full description).

- What is the probability of getting a "1" on the first roll ~~and~~ or a "Four" on the 2nd roll.

These are composite events, and they are not ME. But we can further divide the outcomes to make a situation in which the possible events are ME.

Such as,
$\boxed{A}$ Getting a "1" on the 1st roll (No 4 on 2nd).
$\boxed{B}$ Getting a "4" on the 2nd roll (No 1 on 1st)
$\boxed{C}$ Getting both a 1 and a 4 on the first and second roll, respectively.

$$P_{success} = P_A + P_B + P_C$$

$$P_B = \left(\frac{5}{6}\right)\left(\frac{1}{6}\right) = \frac{5}{36}$$

$$P_A = \left(\frac{1}{6}\right)\left(\frac{5}{6}\right) = \frac{5}{36}$$

$$P_C = \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) = \frac{1}{36}$$

$$\boxed{P_{succes} = \frac{11}{36}}$$

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Correlated Events (CE) - Events are correlated if the outcome of one event depends on the outcome of another event.

Example | Three balls are in a barrell. R G G

$P_{G, 1^{st}} = \frac{2}{3}$

If a green is removed, then R G

$P_{G, 2^{nd}} = \frac{1}{2}$

Conditional Probability

What is the probability of getting snow given that snow only occurs when dark clouds are seen.

A ≡ Dark Sky

B ≡ Snow

Correlated Events

$P(B|A)$ Probability of observing B given that A has occured.

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Joint Probability - $P(A \cap B)$

$P(AB)$

Probability that both A <u>and</u> B occur.

<u>General Multiplication Rule (Bayes Rule)</u>

$$p(AB) = p(B|A)\,p(A)$$

If A and B are not correlated, then

$$p(B|A) = p(B)$$

This would mean that (in the case of the snow) the probability of getting snow does not depend on the observation of dark clouds.

$p(B|A)$ "a posterori" probability

$p(B)$ "a priori" probability.

$$p(G_1) = \frac{2}{3}$$

$$p(G_2|G_1)\,p(G_1) = \left(\frac{1}{2}\right)\left(\frac{2}{3}\right) = \frac{1}{3}$$

Probability of getting G on 1st and G on 2nd

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P of Red on 3rd, given that G was drawn on 2nd and 1st:

$$P(R_3 \text{ with } G_2 \text{ on 2nd and } G_1 \text{ on 1st}) \Rightarrow P(G_1 G_2 R_3)$$

$$= P(R_3 | G_2 G_1)\, P(G_2 | G_1)\, P(G_1)$$

$$= (1) \left(\tfrac{1}{2}\right) \left(\tfrac{2}{3}\right) = \tfrac{1}{3}$$

General Addition Rule

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$\cup$ - Union with

$\cap$ - Intersection of

When events/outcomes are ME, they can never occur at the same time and thus can never intersect ($\cap$).

$$P(A \cap B) = 0 \quad \text{when } A, B \text{ are ME.}$$

Thus, the above would reduce to the old Addition Rule when these events are ME.

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If events are Independent (IE)

$$p(A \cap B) = p(AB)$$

Degree of Correlation

$$g \equiv \frac{p(B|A)}{p(B)} = \frac{p(B|A)\,p(A)}{p(B)\,p(A)} = \frac{p(AB)}{p(B)\,p(A)}$$

If $g > 1$ positive correlation

$g < 1$ negative correlation

$g = 0$ ~~Zero correlation~~. A occurs, B cannot occur. (Mutually Exclusive Events)

$g = 1$ there is no correlation.

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Correlations where position matters:

(1) P_{HTHH} in 4 coin flips.

$$P_{HTHH} = p_H^{n_H} \cdot p_T^{(N-n_H)}$$

If $n_H = 3$

$N - n_H = n_T$

$N = n_H + n_T$