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Course 565/665 Lecture Number ______ Date 1/30/03

Lecturer Dr. Silvia Cavagnero Note Taker Eric Fulmer

Probability Distribution Functions

$$p(n,N) = p^N (1-p)^{(N-n)} \frac{N!}{n!\,(N-n)!}$$ [Binomial Dist. Fn.]

N = # of events

* A collection of elementary events which are
  1. ME
  2. Have a Yes/No type possibility.
* The Individual Events (Elementary Events) must be Independent Events.

If this is indeed a probability distribution, then

$$\sum_{n=0}^{N} p(n,N) = \sum_{n=0}^{N} p^n (1-p)^{(N-n)} \frac{N!}{n!\,(N-n)!}$$

$$= (1-p)^N + Np(1-p)^{(N-1)} + \frac{N(N-1)}{2} p^2 (1-p)^{(N-2)} + \dots$$

$$+ Np^{(N-1)}(1-p) + p^N = (p + (1-p))^N = 1^N = 1$$

In fact,

$$\sum_{n=0}^{N} p(n,N) = 1$$

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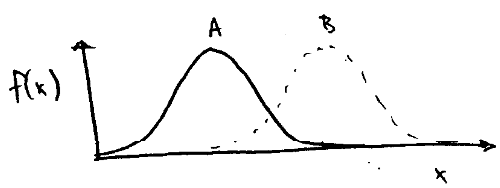
Polynomial (or Multinomial) Distribution Function

- In this case, there are more than 2 possible outcomes.

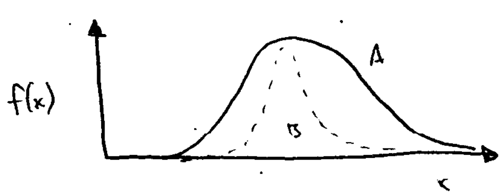
$$P(n_1, n_2, \ldots, n_t; N) = p_1^{n_1} p_2^{n_2} \cdots p_t^{n_t} \frac{N!}{n_1! n_2! \cdots n_t!}$$

t represents the number of outcomes possible. Note that when $t=2$, the above equation reduces down to the Binomial Distribution.

Averages and Standard Deviations



$\bar{x}_A < \bar{x}_B$ (Averages)



σ^2 is the variance

σ is the standard deviation.

$\sigma_B^2 < \sigma_A^2$ since A's distribution is broader.

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Moments of a Function

0^{th} Moment has no physical meaning $(=1)$.

1^{st} Moment $\longrightarrow$ The Average

2^{nd} Moment $\longrightarrow$ Related to the Variance

$$\langle x^n \rangle = \underbrace{\int_a^b x\, p(x)\, dx}_{\text{(Normalized)}} = \frac{\int_a^b x\, \psi(x)\, dx}{\int_a^b \psi(x)\, dx}$$

$$\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2$$

Variance is the 2^{nd} moment minus the 1^{st} moment (the average) squared.

- Standard Deviation is the square root of the Variance.

Continuous Distributions: $\langle x \rangle = \int_a^b x\, p(x)\, dx$

Discrete Distributions: $\langle i \rangle = \sum_{i=1}^{t} i$ [redacted] $p_i = \sum_{i=1}^{t} i p_i$

$p(x)$ or [redacted] p_i are the weighting coefficients.

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Properties of Averages

① $\langle a f(x) \rangle = a \int_a^b f(x)\, p(x)\, dx = a \langle f(x) \rangle$

② $\langle f(x) + g(x) \rangle = \langle f(x) \rangle + \langle g(x) \rangle$

Variance (σ^2)

$$\sigma^2 = \langle (x - \langle x \rangle)^2 \rangle = \langle (x-a)^2 \rangle = \langle x^2 + 2xa + a^2 \rangle$$

$$= \langle x^2 \rangle + \langle 2xa \rangle + \langle a^2 \rangle = \langle x^2 \rangle + 2a \langle x \rangle + a^2$$

$$= \langle x^2 \rangle - 2 \langle x \rangle \langle x \rangle + \langle x \rangle^2 = \langle x^2 \rangle - 2 \langle x^2 \rangle + \langle x \rangle^2$$

$$\boxed{\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2}$$

In the above example $\langle x \rangle = a$ since this quantity is a constant for a given data set.

Exponential Distribution Function

$\Psi(x) = e^{-ax}$ $\quad 0 < x < \infty$

$$\Psi_0 = \int_0^\infty e^{-ax} = \frac{-1}{a} e^{-ax} \Big|_0^\infty = \frac{-1}{a}(0 - 1) = \frac{1}{a}$$

$p(x) = a\, e^{-ax}$ [Normalized Exponential (Boltzmann) Distribution]

$$\langle x \rangle = \int_0^\infty x\, p(x)\, dx = a \int x\, e^{-ax}\, dx = -e^{-ax}\left(x + \frac{1}{a}\right)\Big|_0^\infty = \frac{1}{a}$$