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Lecturer Dr. Silvia Cavagnero Note Taker Eric Fulmer

## Clarifications

$\sigma^2$ is defined as the variance. $\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2$

$\sigma$ is the standard deviation; $\sqrt{\text{variance}}$

$\sigma = \sqrt{\sigma^2}$

$\langle x^2 \rangle$ is the second moment, not the standard deviation.

## Integration by Parts

$u = u(x)$
$v = v(x)$

$$\int u\, dv = uv - \int v\, du$$

$\boxed{\text{Informal HW}}$ Determine $\sigma^2$ for the Boltzmann (Exponential) Distribution.

## Chapter Two

Extremum Principles and Equilibria

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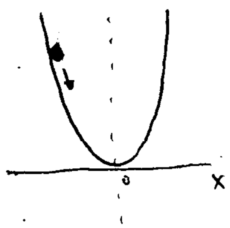


$V = mgz$

$F = -mg$

$V$ is the potential energy.
$z$ is the height
$m$ is the mass
$g$ is the gravitational constant
$F$ is a force.

The force is always the negative derivative of the potential with respect to the independent variable.

$$F = -\left(\frac{dV}{dx}\right)$$

In the quadratic potential above, we can substitute in $z = x^2$. Thus, the potential is

$V = mgx^2$.

When the force on the object is zero, the object has reached equilibrium.

$$f = -\left(\frac{dV}{dx}\right)_{x = x_{minimum}} = 2mgx_{minimum} = 0$$

Thus, $x_{minimum} = 0$.

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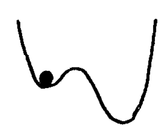
This makes intuitive sense because the minimum of the potential energy curve on the previous page was the bottom, or the point where $x = 0$. Systems always tend towards equilibrium given enough time.

## Newtonian Forces

$$F = ma = -\left(\frac{dV(x)}{dx}\right)$$

## Types of Equilibria

- Stable if $V = V_{min}$ 
- Neutral if $V =$ flat 
- Metastable if V is at a minimum, but there are lower minima elsewhere. 

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There are also <u>Unstable</u> situations.

If $V = V_{max}$, $\frac{dV}{dx} = 0$. (true for $x = x_{min}$ and $x = x_{max}$)

$$V(x) - V(x_{max}) < 0$$

<u>Stable</u> for $x \neq x_{min}$ $\quad V(x) - V(x_{min}) > 0$

<u>Neutral</u> for any $x$ $\quad \frac{dV}{dx} = 0$

<u>Metastable</u> for $|x - x_{min}| =$ small, $V(x) - V(x_{min}) > 0$

but for $|x - x_{min}| =$ large, $V(x) - V(x_{min}) < 0$

Probability revisited.

Coin Flips: $N = 4$

| $n_H$ | $W$ | $\ln W$ |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 4 | |
| 2 | 6 | |
| 3 | 4 | |
| 4 | 1 | 0 |

total $W = 16$

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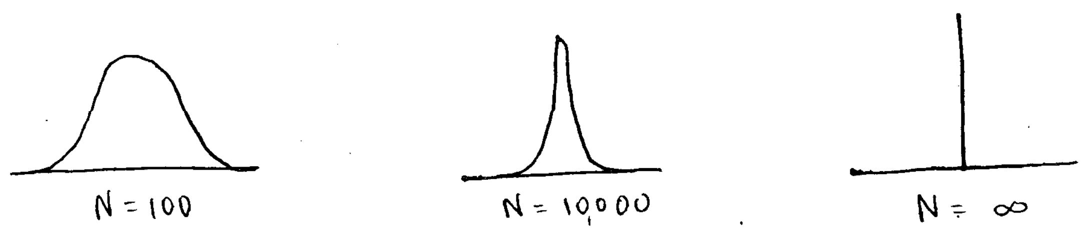
Note However,

As $N \longrightarrow \infty$ [ie. as N becomes a very large number], the probability tends towards on value [the width of the distribution tends towards zero].




<u>Extremium Principle #1</u> - Systems tend toward equilibrium (where external forces equal zero).

<u>Extremum Principle #2</u> - Systems tend toward a state of maximum multiplicity.

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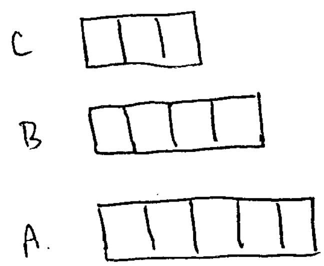
Consider a 1D ~~space~~. lattice space.

Example: Why do gases exert pressure?




M = # of lattice sites.

$M_C = 3$

$M_B = 4$

$M_A = 5$

Let there be two situations:

Occupied by a particle

Unoccupied

Let $n = 3$ (there are 3 particles).

$$W = \frac{M!}{n!\,(M-n)!}$$

$$W_A = \frac{5!}{3!\,2!} = \frac{20}{2} = 10$$

$$W_B = \frac{4!}{3!\,1!} = \frac{4}{1} = 4$$

$$W_C = \frac{3!}{3!\,0!} = \frac{1}{1} = 1$$

More possiblities for A than C.