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Course 565/665 Lecture Number ____ Date 2/6/02

Lecturer Eric Fulmer Note Taker Andrea

Sequence: A set of quantities ordered by integrers.

Series: A sum of the sequence.

- Infinite
- Partial

Type of series

1. Power Series
   - Geometric
   - Arithmetic - Geometric
2. Taylor Series.

Power Series

$$f(x) = \sum_{k=0}^{\infty} c_k x^k$$

$$= c_0 + c_1 x + c_2 x^2 + \dots$$

Geometric

$$S_n = a + ax + ax^2 + \dots ax^{n-1}$$

$$S_n = \frac{a(1-x^n)}{(1-x)} \qquad \text{if } x \neq 1$$

Example 4.1

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$$

$a = 1$

$x = \frac{1}{2}$

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$$\boxed{S_\infty = \sum_{k=1}^{\infty} a x^{k-1} = \frac{a}{1-x}}$$

$$S_\infty = \frac{a}{1-x} = \frac{1}{1-1/2} = 2$$

Arithmetic - Geometric Series.

$$t_n = \sum_{k=1}^{n} a k x^k$$

$$\frac{dS_\infty}{dx} = a + 2ax + 3ax^2 + \dots$$

$$x\frac{dS_\infty}{dx} = t_\infty$$

$$S_\infty = \frac{a}{(1-x)}$$

$$t_\infty = x\frac{d}{dx}\left(\frac{a}{(1-x)}\right)$$

$$= ax\frac{d}{dt}(1-x)^{-1}$$

$$\boxed{t_\infty = \frac{ax}{(1-x)^2}}$$

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$$Q(x) = 1 + x + x^2 + \dots + x^n$$

$$= \sum_{i=0}^{n} x^i$$

$$\langle i \rangle = \frac{\sum_{i=0}^{n} i\,Q(x)}{\sum_{i=0}^{n} Q(x)} = \frac{x + 2x^2 + 3x^3 + \dots + nx^n}{1 + x + x^2 + \dots + x^n}$$

$n \to \infty$

$$\langle i \rangle = \frac{t_\infty}{S_\infty} = \frac{ax/(1-x)^2}{a/(1-x)} = \frac{x}{(1-x)}$$

<u>Ex. 4.2</u>

P is the probability that a monomer has reacted and connected to the polymer.

$$n_k = p^{k-1}(1-p)$$

$$P(k) = \frac{n_k}{\sum_{k=1}^{\infty} n_k} \qquad \langle k \rangle \; ?$$

$$\langle k \rangle = \sum_{k=1}^{\infty} k\,P(k)$$

$$= \frac{p}{p} \frac{\sum_{k=1}^{\infty} k\,p^{k-1} \cancel{(1-p)}}{\sum_{k=1}^{\infty} p^{k-1} \cancel{(1-p)}} = \frac{\frac{1}{p} \sum_{k=1}^{\infty} k\,p^k}{\sum_{k=1}^{\infty} p^{k-1}} = \frac{\frac{1}{p} t_\infty}{S_\infty} = \frac{1}{1-p}$$

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Taylor Series

Expanding f(x) about x=a

$$f(x) = f(a) + f^{(1)}(a)\cdot(x-a) + \frac{f^{(2)}(a)}{2!}(x-a)^2 + \dots$$

$$= \sum_{n=0}^{\infty} \frac{1}{n!} \frac{d^n}{dx^n} f(x)\Big|_{x=a} (x-a)^n$$

Ex. 4.3 Taylor Series of $f(x) = e^{-bx}$ about $x=0$

- $f(a) = e^{-b\cdot 0} = 1$
- $\frac{1}{1!} f^{(1)}(a)\cdot(x-a)^1 = 1 - b\cdot e^{-bx}\big|_{x=0} = -b\cdot x$
- $\frac{1}{2!} f^{(2)}(a)\,(x-a)^2 = \frac{1}{2}\cdot b^2 e^{-bx}\Big|_{x=0} (x-0)^2 = b^2x^2$

$$f(x) = e^{-bx} \approx 1 - bx + \frac{b^2x^2}{2}$$

Stirling's approximation

$$\ln n! \approx n\ln n - n$$