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Course 565/665 Lecture Number ______ Date 2/7/03

Lecturer Eric Fulmer Note Taker Fulmer

Random Walks in biology

- Diffusion of particles
- Conformations of polymer chains
- Conduction of heat.

1D Random Walk.

Two choices: Steps with length $+x$, $-x$

Random, so equally probable. Independent.

N total Steps

m in the $+x$ direction

$(N-m)$ in the $-x$ direction

What is $P(m,N)$?

$$P(m,N) = \left(\frac{1}{2}\right)^m \left(\frac{1}{2}\right)^{N-m} \frac{N!}{m!\,(N-m)!}$$

$$= \left(\frac{1}{2}\right)^N \frac{N!}{m!\,(N-m)!}$$

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We aim to approximate P. Factorials are difficult to work with. Log (Factorial) is much better.

Taylor Series

$$\ln P(m) = \ln P(m^*) + \left(\frac{d \ln P}{dm}\right)_{m^*} (m - m^*)$$

$$+ \frac{1}{2}\left(\frac{d^2 \ln P}{dm^2}\right)_{m^*} (m - m^*)^2 + \ldots$$

What is $\ln P$? $\boxed{\ln N! = N \ln N - N}$

$$\ln P = N \ln N - \cancel{N} - m \ln m + \cancel{m} - (N-m)\ln(N-m) + \cancel{N} - \cancel{m}$$

$$+ N \ln \frac{1}{2}$$

We want $m = m^*$ in which $P(m)$ or $\ln P(m)$ is a maximum which $\left(\frac{d \ln P}{dm}\right)_{m^*} = 0$.

$$\left(\frac{d \ln P}{dm}\right)_{m^*} = -\ln m^* - 1 + \ln (N - m^*) + 1 = 0$$

$$= -\ln\left(\frac{m}{N - m^*}\right) = 0$$

$$m^* = \frac{N}{2}$$

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$$\left(\frac{d^2 \ln P}{dm^2}\right)_{m^*} = \left(-\frac{1}{m} - \frac{1}{N-m}\right)_{m^* = \frac{N}{2}} = \frac{-4}{N}$$

Thus,

$$\ln P = \ln P(m^*) + 0 + \frac{1}{2} \cdot \left(-\frac{4}{N}\right) \cdot \left(m - \overset{m^*}{\frac{N}{2}}\right)^2$$

$$P = P(m^*) \cdot e^{-2(m-m^*)^2/N}$$

What is the net forward progress? (x)

$$x = m - (N-m) = 2m - N$$

$$x^* = 2m^* - N = 0$$

$$x = 2m - N \longrightarrow m = (x+N)/2 \quad , \quad m^* = N/2$$

$$P(x) = p^* \quad e^{-x^2/2N}$$

p^*? Normalize!

$$\int_{-\infty}^{\infty} P(x)\, dx = \int_{-\infty}^{\infty} p^* \; e^{-x^2/2N}\, dx = 1$$

$$p^* = \frac{1}{\sqrt{2\pi N}}$$

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$$P(x) = (2\pi N)^{-1/2} \; e^{-x^2/2N}$$

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 \, P(x) \, dx$$

$$= \left(\frac{1}{2\pi N}\right)^{1/2} \int_{-\infty}^{\infty} x^2 \, e^{-x^2/2N} \, dx$$

$$= \left(\frac{1}{2\pi N}\right)^{1/2} \cdot N \, (2\pi N)^{1/2}$$

$$\langle x^2 \rangle = N$$ variance

$$\langle x^2 \rangle^{1/2} = \sqrt{N}$$ st. dev.

root mean squared displacement

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Solutions to the Sample Problems

① $\exp(x)$

$$1 + \frac{x}{1} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots \qquad \boxed{\text{let } x = 1}$$

term	sum
1	1
2	2
⋮	⋮
7	2.7164

$\boxed{e = 2.7183}$

②

$f(0) = \cos 0 = 1$

$f^{(1)}(0) = -\sin x\big|_0 = 0$

$\frac{1}{2} f^{(2)}(0)\, x^2 = \frac{1}{2} x^2 \cdot -\cos x = -\frac{1}{2} x^2$

$f^{(3)}(0) = 0$

$f^{(4)}(0) = +\frac{x^4}{4!}$

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(3) $$\frac{dp}{dt} = \alpha p$$

$$\int_0^t \frac{dp}{p} = \int_0^t \alpha \, dt$$

$$\ln\left(\frac{p(t)}{p_0}\right) = \alpha t$$

$$p(t) = p_0 e^{\alpha t}$$

(4) Expand e^{-bx} about $x = 5$ to the second order.
Taylor Series Expansion.

$$e^{-bx} = e^{-5b} + -b e^{-5b}(x-5) + \frac{b^2}{2} e^{-5b}(x-5)^2$$

✓