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Course 565/665 Lecture Number ______ Date 1/28/03

Lecturer Dr. Silvia Cavagnero Note Taker Eric Fulmer

LAST TIME: HTHH

① Situations where Sequence Matters

$$P_{seq} = p_H^{n_H} p_T^{(N-n_H)} = \left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^1 = \frac{1}{16}$$

② Situations where Composition Matters.

3 heads, 1 tail. (HHHT HHTH HTHH THHH).

$$P_{comp} = P_{seq} \times W \quad \text{where } W \text{ is the Multiplicity.}$$

$$= p_H^{n_H} \cdot p_T^{(N-n_H)} \frac{N!}{n_H!\,(N-n_H)!}$$

The heads are not distinguishable between themselves, so if we multiplied by $N!$, we would be over-counting by a factor of six. We must divide by the number of H's occuring factorial $(n_H!)$ and the number of T's occuring factorial $[(N-n_H)!]$.

Thus,

$$P_{comp} = \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right) \frac{4!}{3!\,1!} = \frac{1}{16} \cdot 4$$

$$= \frac{4}{16}$$ This agrees with the above.

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Example | What is the probability of drawing letters out of a barrel and drawing them in the order that they exist is the actual alphabet.

When drawing out the 1st letter, there are 26 possibilities. The 25 possibilities. etc....

$$P_{ABC\ldots YZ} = \frac{1}{W} = \frac{1}{N!} = \left(\frac{1}{26}\right)\left(\frac{1}{25}\right)\left(\frac{1}{24}\right)\cdots\left(\frac{1}{1}\right)$$

This is the case because the letters are all distinguishable.

Distinguishable vs. Indistinguishable Events:

Multiplicity of $A_1 A_2 H$ vs. AAH

$$W_{distinguishable} = 3! = 6$$

$$W_{distinguishable} = \frac{N!}{n_{A_1}!\, n_{A_2}!\, n_H!} = \frac{3!}{1!\,1!\,1!} = 6$$

$$W_{indistinguishable} = \frac{N!}{n_A!\, n_H!} = \frac{3!}{2!\,1!} = \frac{3\cdot 2\cdot 1}{2\cdot 1} = 3$$

This accounts for the overcounting.

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The above describes events with 2 possibilities ($W_{indistinguishable}$). If there are more than 2 possibilities we do the same concept but apply it for all of the possibilities.

$$W = \frac{N!}{n_A!\, n_B! \cdots n_M!}$$

where $A, B, \ldots, M$ are all of the distinguishable possible outcomes and $n_A, n_B, \ldots n_M$ are the # of each outcome.

Ex) Multiplicity of arranging F, R, E, E, Z, E, R.

$$W = \frac{7!}{1!\,3!\,2!\,1!} = \frac{N!}{n_F!\, n_E!\, n_R!\, n_Z!} = \frac{7 \cdot 6 \cdot 5 \cdot 4}{2}$$

If the letters are all distinguishable $(F, R_1, E_1, E_2, Z, E_3, R_2)$, then the multiplicity is

$$W = \frac{7!}{1! \cdots 1!} = 7!$$

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Probability Distribution Functions

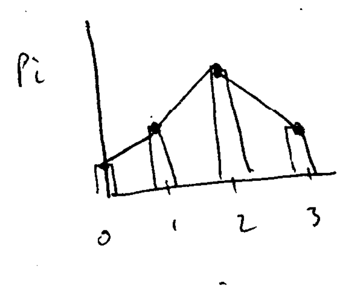
- Describes collection of P's
- Allow mapping ("plotting") P distributions

* Events need to be ME.
* Map a collectively exhaustive (CE) set of events "t".

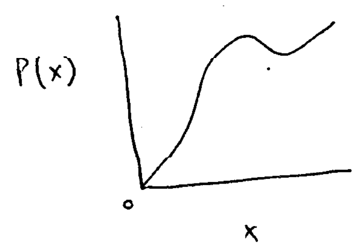
$\boxed{\text{Notation Note}}$ The summation, $\sum$. This adds up all of the possibilities given from the number on the bottom to the number on the top.

$$\sum_{i=1}^{6} i M^i = 1 \cdot M^1 + 2 \cdot M^2 + 3M^3 + 4M^4 + 5M^5 + 6M^6$$

Discrete Probability Fn.



Continuous Prob. Fn.



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When going from a to b, we must know the "stepping" quantity Δx between points for the discrete p_i. Determining the probability of something occuring between a and b is determined by a summation.

$$P = \sum_{i=a}^{b} p_i \, \Delta x$$

For continuous functions, Δx becomes infinitesimally small and we integrate between a and b.

$$P = \int_a^b p(x)\, dx.$$

Sometimes these probability distribution functions are not normalized (that is, the sum of all possible probabilities is equal to "1"). We must then divide a factor which makes that the case.

$$p(x) = \frac{\Psi(x)}{\Psi_0} \qquad \text{where } \Psi_0 = \int_a^b \Psi(x)\, dx$$

When normalized, $\int_a^b p(x)\, dx = 1$

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Binomial Distribution

● O or H T

- Applies to Y/N kind of events
- ME events

$$P(n, N) = p^n (1-p)^{(N-n)} \frac{N!}{n!\,(N-n)!}$$