A. Binomial series, useful for the most probable distribution.

$$(1 \pm p)^n = 1 \pm np + \frac{n(n-1)p^2}{2!} \pm \frac{n(n-1)(n-2)p^3}{3!} + \frac{n(n-1)(n-2)(1-3)p^4}{4!} \pm \ldots, \quad (p^2 < 1)$$

$$(1 \pm p)^{-n} = 1 \mp np + \frac{n(n+1)p^2}{2!} \mp \frac{n(n+1)(n+2)p^3}{3!} + \frac{n(n+1)(n+2)(1+3)p^4}{4!} \pm \ldots, \quad (p^2 < 1)$$

$$(1-p)^{-1} = 1 + p + p^2 + p^3 + p^4 + p^5 + \ldots$$

$$(1-p)^{-2} = 1 + 2p + 3p^2 + 4p^3 + 5p^4 + 6p^5 + \ldots$$

$$(1-p)^{-3} = 1 + 3p + 6p^2 + 10p^3 + 15p^4 + 21p^5 + \ldots$$

$$(1-p)^{-4} = 1 + 4p + 10p^2 + 20p^3 + 35p^4 + 56p^5 + \ldots$$

$$(1-p)^{-5} = 1 + 5p + 15p^2 + 35p^3 + 70p^4 + 126p^5 + \ldots$$

$$\sum_{x=1}^{\infty} x^2 p^{x-1} = 1 + 4p + 9p^2 + 25p^3 + \ldots, \quad (p^2 < 1)$$

$$= \frac{1 + p}{(1 - p)^3}$$

$$\sum_{x=1}^{\infty} x^3 p^{x-1} = 1 + 8p + 27p^2 + 64p^3 + \ldots, \quad (p^2 < 1)$$

$$= \frac{1 + 4p + p^2}{(1 - p)^4}$$

B. Random variable, the distribution function of the random variable, moments and moments generating function.

random variable : N

sample space of random variable: $1 \le N \le \infty$

distribution function of N: F_N

kth moment of F_N : $m_k \equiv \sum_{N=1}^{\infty} N^k \cdot F_N$

moments generating function: $g(N,s) \equiv \sum_{N=1}^{\infty} s^N \cdot F_N$ such that $m_k = \left[\left(s \cdot \frac{d}{ds} \right)^{(k)} g(N, s) \right]_{s=1}$

If $F_N = p^{N-1} \cdot (1-p)$, $g(N, s) = \sum_{N=1}^{\infty} s^N \cdot F_N = \sum_{N=1}^{\infty} s^N \cdot (1-p) \cdot p^{N-1} = \frac{s(1-p)}{(1-ps)}$

k	$\left(s\frac{d}{ds}\right)^{(k)} \cdot g(N,s)$	m_k
0	$\frac{s(1-p)}{(1-ps)}$	1
1	$\frac{s(1-p)}{(1-ps)^2}$	$(1-p)^{-1}$
2	$\frac{s(1-p)}{(1-ps)^3} \cdot (1+ps)$	$\frac{1+p}{(1-p)^2}$
.		
.		
.		
k	$\frac{s(1-p)}{(1-ps)^{k+1}} \cdot \left[\sum_{x=1}^{k} A_x \cdot (ps)^{x-1} \right]$	$\frac{1+A_2 p + A_3 p^2 + \ldots + A_k p^{k-1}}{(1-p)^k}$ $\approx \frac{k!}{(1-p)^k} = k! m_1^k$