Solutions to Problem Set #10
due 12/04/02

A(6.8) $\Sigma_2 = \dfrac{[1-p^2(f-1)]p}{(1-p)[1-p(f-1)]^3}$, $\dfrac{\partial p}{\partial x} = \dfrac{p(1-p)}{x[1-p(f-1)]}$

where $x \equiv p(1-p)^{f-2}$

$$\Sigma_3 = x\frac{\partial \Sigma_2}{\partial x} = x\cdot\frac{\partial \Sigma_2}{\partial p}\cdot\frac{\partial p}{\partial x}$$

$$= \frac{x p(1-p)}{x[1-p(f-1)]}\frac{\partial}{\partial p}\left\{\frac{[1-p^2(f-1)]p}{(1-p)[1-p(f-1)]^3}\right\}$$

$$= \frac{p\{[1-p^2(f-1)]^2 + 2p(f-1)(1-p)^2\}}{(1-p)[1-p(f-1)]^5}$$

$$\therefore N_z = \frac{\Sigma_3}{\Sigma_2} = \frac{\cancel{p}\{[1-p^2(f-1)]^2+2p(f-1)(1-p)^2\}}{\cancel{(1-p)}[1-p(f-1)]^{\cancel{5}2}}\cdot\frac{\cancel{(1-p)}[1-p(f-1)]^{\cancel{3}}}{[1-p^2(f-1)]\cancel{p}}$$

$$= \frac{[1-p^2(f-1)]^2+2p(f-1)(1-p)^2}{[1-p(f-1)]^2[1-p^2(f-1)]}$$

$$\frac{N_z}{N_w} = \frac{[1-p^2(f-1)]^2+2p(f-1)(1-p)^2}{\cancel{[1-p(f-1)]^2}\cdot[1-p^2(f-1)]}\cdot\frac{\cancel{[1-p(f-1)]^2}}{[1-p^2(f-1)]}$$

$$= 1+\frac{2p(f-1)(1-p)^2}{[1-p^2(f-1)]^2}$$

$$\lim_{p\to p_c}\frac{N_z}{N_w} = 1+\frac{2\cancel{\left(\frac{1}{f-1}\right)}\cancel{(f-1)}(1-p_c)^2}{[1-p_c\cancel{\left(\frac{1}{f-1}\right)}\cdot\cancel{(f-1)}]^2} = 1+2 = 3$$

$$p_c = \frac{1}{f-1}$$

B(6.11) $f=4$, $p_c = \dfrac{1}{f-1} = \dfrac{1}{3}$

$$P_{sol}^{1/4} = 1-p+p\cdot P_{sol}^{3/4}$$

$$P_{sol}^{1/4} \equiv x$$

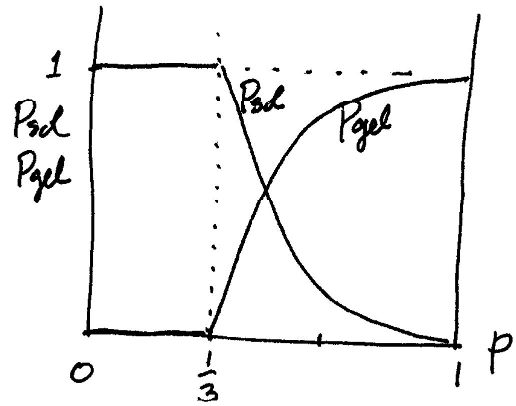
$x = 1-p+p\cdot x^3$, $px^3-x-p+1=0$, $p(x^3-1)-(x-1)=0$

$p(x-1)(x^2+x+1)-(x-1) = (x-1)[p(x^2+x+1)-1] = 0$

$x=1$ + $px^2+px+(p-1)=0$, $x = \dfrac{-p\pm\sqrt{p^2-4p(p-1)}}{2p}$

$\underline{P_{sol}^{1/4}=1,\ p<p_c=\frac{1}{3}}$, $P_{sol} = \left(\dfrac{-p+\sqrt{4p-3p^2}}{2p}\right)^4$, $p>p_c=\dfrac{1}{3}$

SPS10/2



C(6.14)

Lattice	f	$p_c = \frac{1}{f-1}$
i) Honeycomb	3	1/2
ii) Square	4	1/3
iii) triangular	6	1/5
iv) diamond	4	1/3
v) Simple cubic	6	1/5

vi) The gel points are less than those listed in Table 6.1 & the difference arises from the mean-field approach for percolation not taking into account intramolecular bonds.

vii) $d=3$ closer to those in Table 6.1 ($f=6$) because they are less likely for intramolecular bonding

D(6.21) $n(N) \approx N^{-\tau}$, $m_k = \int_0^\infty N^k n(N)\,dN \approx \int_0^\infty N^{k-\tau}\,dN \approx \left.\frac{N^{k-\tau+1}}{k-\tau+1}\right|_0^\infty$

m_k is finite if & only if $k-\tau+1<0$, or $\tau > k+1$

(i) $N_n = m_1/m_0$ to be finite, $\underline{\underline{\tau > 2}}$

(ii) $N_w/N_n = \frac{m_2 \cdot m_0}{m_1^2} = \frac{\int_0^\infty N^{1-\tau}\,dN}{\left(\int_0^\infty N^{2-\tau}\,dN\right)^2}$ is finite if $\underline{\underline{\tau > 3}}$

(iii) $N_z \equiv \frac{m_3}{m_2} = \frac{\int_0^\infty N^{3-\tau}\,dN}{\int_0^\infty N^{2-\tau}\,dN}$ is finite if $\underline{\underline{\tau > 4}}$

(iv) $P_{sol} = m_1$, $\underline{\underline{\tau > 2}}$

E(6.22) $\varepsilon \equiv \frac{p-p_c}{p_c} = -0.01$

(i) $N^* \approx |\varepsilon|^{-1/\sigma} = \frac{1}{(10^{-2})^2} = 10^4$ where $\sigma = 1/2$ mean field theory

(ii) $R_g \approx b(N^*)^{1/4} = 3\mathring{A} \cdot (10^4)^{1/4} = 30\mathring{A}$

(iii) $R \approx b(N^*)^{2/5} \approx 3 \cdot (10^4)^{2/5} \cong 120\mathring{A}$

(iv) $A_e \approx kTv\frac{(N^*)^2}{R^3}$, $A_s \approx kT\frac{R^2}{(N^*)^{1/2}b^2}$) $A \approx kT\left[\frac{R^2}{(N^{*1/4}b)^2} + v\frac{(N^*)^2}{R^3}\right]$

$$\left(\frac{\partial A}{\partial R}\right) = 0 = \frac{2R}{((N^*)^{1/4}b)^2} - \frac{3v(N^*)^2}{R^4}$$

$$\frac{2R}{N^{*1/2}b^2} = \frac{3b^3N^{*2}}{R^4}, \quad R^5 \approx b^5N^{*5/2}, \quad R \approx bN^{*1/2} \approx 3 \cdot 10^2\mathring{A}$$

F(6.28) Ideal random branched chain

$$R_{g,0} \approx bN_0^{1/2}, \quad R_g \approx R_{g,0}\left(\frac{N}{N_0}\right)^{1/4} \approx bN_0^{1/4}N^{1/4}$$

(i) $R_g \approx 5\mathring{A} \cdot (10^2)^{1/4}(10^4)^{1/4} \approx 5 \cdot 10^{3/2} \cong 158\mathring{A}$, $N_0 = 10^2$

(ii) $R_g \approx 5\mathring{A} \cdot 10^{1/4} \cdot 10 \cong 89\mathring{A}$, $N_0 = 10$

(iii) $\phi \cong \left(\frac{10^4}{10^6}\right)^{1/4} \cong 0.32$ (i)

$\phi \cong \left(\frac{10^4}{10^3}\right)^{1/4} \cong 1.78$ (ii)) pervaded volume fractions

$$\phi \approx \frac{Nb^3}{R^3} \approx \frac{N}{(N_0N)^{3/4}} \approx \left(\frac{N}{N_0^3}\right)^{1/4}$$

For this chain $N > N_0^3$ $\phi > 1$ because the fractal dimension of ideal randomly branched polymer is 4, higher than 3-D space.

G(7.1) $dA = -SdT - PdV + fdL$, $\left(\frac{\partial S}{\partial L}\right)_{T,V} = -\left(\frac{\partial f}{\partial T}\right)_{V,L}$

$\left(\frac{\partial S}{\partial V}\right)_{T,L} = \left(\frac{\partial P}{\partial T}\right)_{V,L}$, $\left(\frac{\partial P}{\partial L}\right)_{T,V} = -\left(\frac{\partial f}{\partial V}\right)_{T,L}$

H(7.7) $E = 2(1+\nu)G = 3(1-2\nu)K$

(i) $E = 3(1-2\nu)K$, $\nu = \frac{1}{2}\left[1 - \frac{E}{3K}\right] = \frac{1}{2}\left(1 - \frac{1.3\cdot 10^6}{3.2\cdot 10^9}\right)$
$= 0.4999$

(ii) $G = \frac{E}{2(1+\nu)} = \frac{1.3\cdot 10^6\,Pa}{2(1.4999)} \cong 4.334\cdot 10^5\,Pa$

(iii) $\frac{G}{K} = \frac{3}{2}\left(\frac{1-2\nu}{1+\nu}\right)$, if $K >> G$ $\underline{1-2\nu \approx 0}$
or $\nu = \frac{1}{2}$

(iv) $E = 3(1-2\nu)K$, $E/K = 3 - 6\nu$

$\nu = \frac{1}{2} - \frac{E}{6K}$, hence $\frac{E}{6K}$ is the correction factor for a compressible body $\nu < \frac{1}{2}$

I(7.9)

$\cancel{m} g\Delta h = \cancel{m} C_p \Delta T$

(i) $\Delta T = \frac{g\Delta h}{C_p} \cong \frac{9.81\,\frac{m}{s^2}\cdot 0.08m}{2200\,J/kg\cdot K} \cong 3.57\cdot 10^{-4}\,K$

$T + \Delta T = 300.150357\,K$

(ii) $\frac{\Delta E}{E} = \frac{\Delta T}{T} = \frac{3.57\cdot 10^{-4}}{3\cdot 10^2} \cong 1.19\cdot 10^{-6}$ hardly a change!