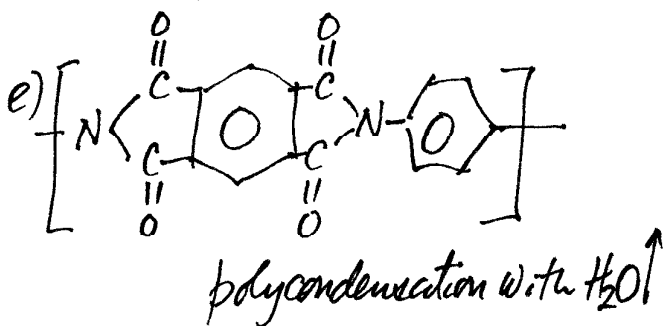
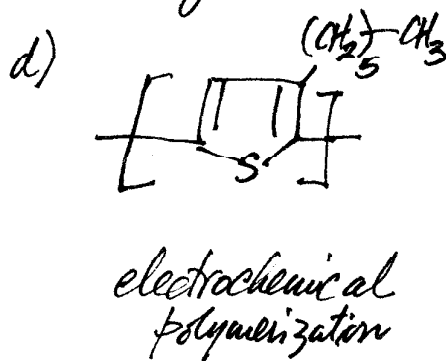
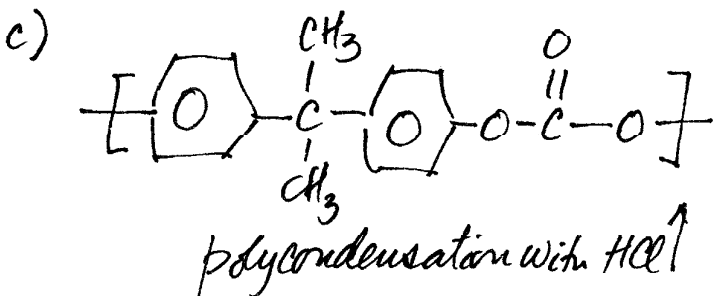
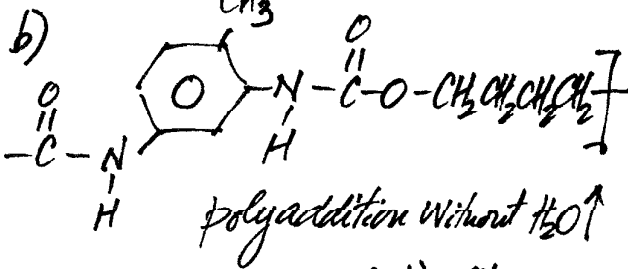
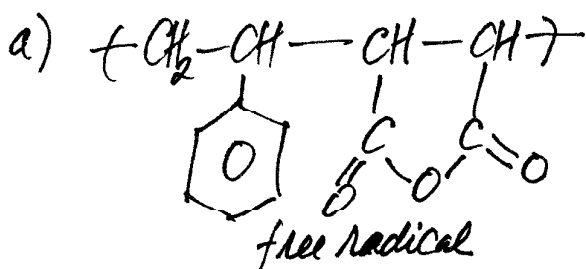


CHEM 654 Problem Set #1 Solutions

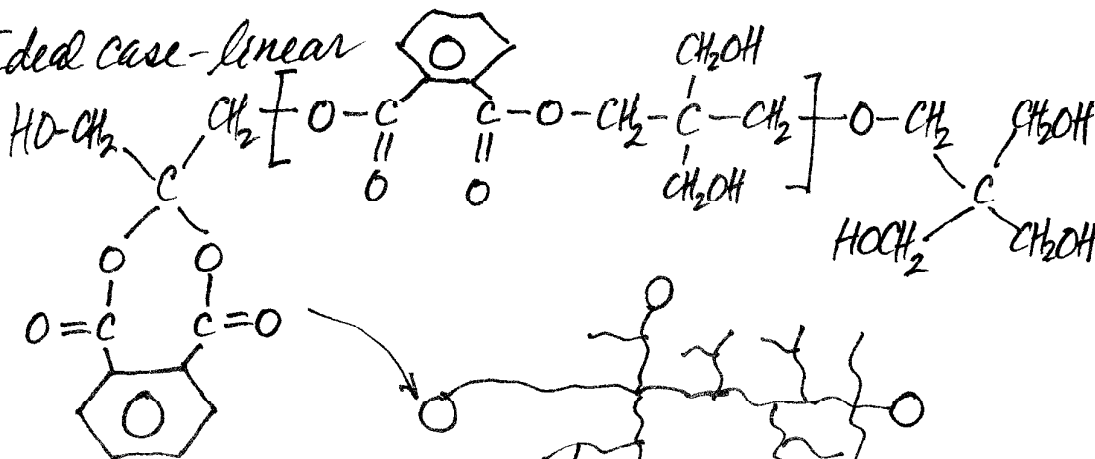
due 02/03/03

1.

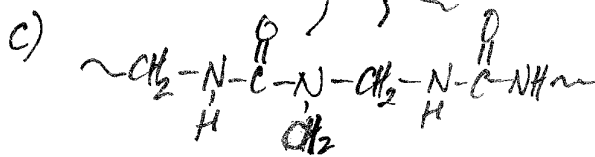


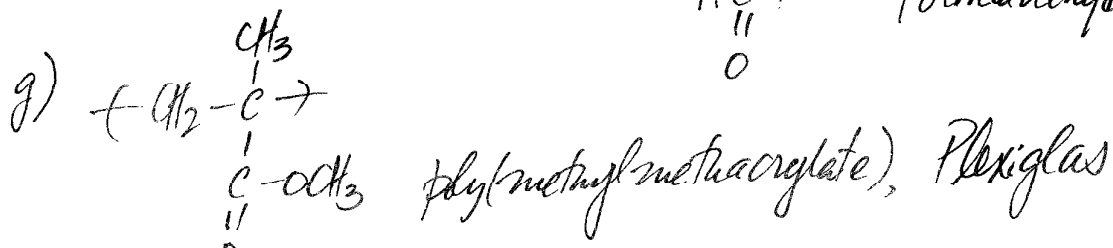
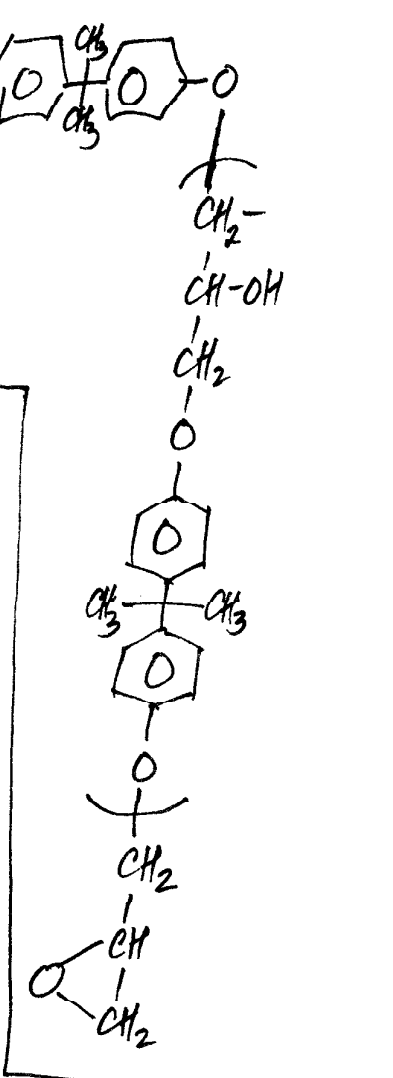
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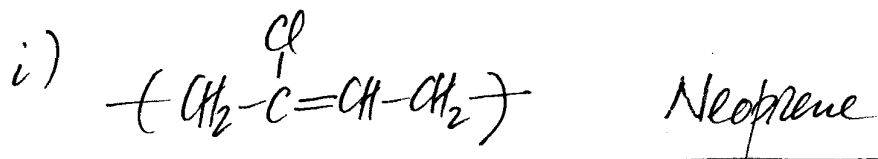
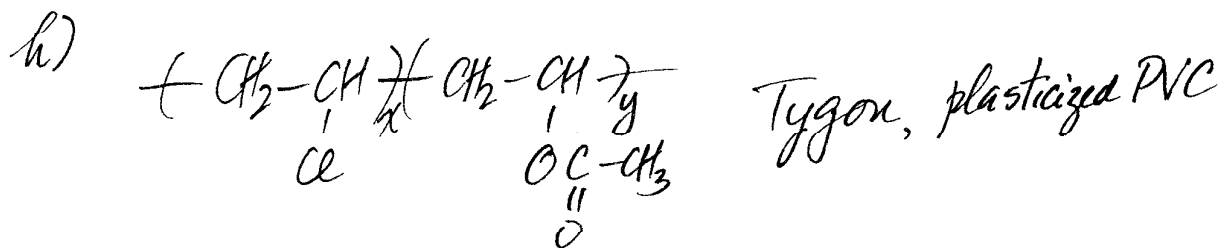
a) Ideal case - linear



b) See p.48







4. a) $M_n = \frac{\sum w_i}{\sum (w_i/M_i)} = \frac{11 \cdot 10^3}{10+0.1} = 1.1 \cdot 10^3 \text{ (g/mol)}$

$M_w = \frac{\sum w_i M_i}{\sum w_i} = \frac{(10 + 10) \cdot 10^3}{11} = 1.8 \cdot 10^3 \text{ (g/mol)}$

$M_z = \frac{\sum w_i M_i^2}{\sum w_i M_i} = \frac{10(10^3)^2 + 1(10^4)^2}{2 \cdot 10^4} = 5.5 \cdot 10^3 \text{ (")}$

$M_{z+1} = \frac{\sum w_i M_i^3}{\sum w_i M_i^2} = \frac{10(10^3)^3 + 1(10^4)^3}{1.1 \cdot 10^8} = 9.2 \cdot 10^3 \text{ (")}$

b) $M_n = \frac{6 \cdot 10^4}{3+1+\frac{1}{3}} = 1.4 \cdot 10^4 \text{ (g/mol)}$

$M_w = 10^4 \left(\frac{3+4+3}{6} \right) = 1.7 \cdot 10^4 \text{ (")}$

$M_z = \frac{3(10^4)^2 + 2(2 \cdot 10^4)^2 + 1(3 \cdot 10^4)^2}{10^5} = \frac{10^8}{10^5} (3+8+9) = 2 \cdot 10^4 \text{ (g/mol)}$

5. a) $\sum x = \frac{x W_x}{\sum x W_x} = \frac{x^2 p^{x-1} (1-p)^2}{\frac{(1+p)}{(1-p)}} = x^2 p^{x-1} (1-p)^3 / (1+p)$

$\sum x W_x = (1-p)^2 \sum_{x=1}^{\infty} x^2 p^{x-1} = (1-p)^2 \frac{1+p}{(1-p)^3} = \frac{1+p}{1-p}$

$\sum_{x=1}^{\infty} W_x = \sum_{x=1}^{\infty} x p^{x-1} (1-p)^2 = (1-p)^2 (1 \cdot p^0 + 2p + 3p^2 + \dots) = (1-p)^2 \frac{1}{(1-p)^2} = 1 \text{ (normalized!)}$

$\sum_{x=1}^{\infty} Z_x = \frac{(1-p)^3}{(1+p)} \sum_{x=1}^{\infty} x^2 p^{x-1} = \frac{(1-p)^3}{(1+p)} (1 \cdot p^0 + 4p + 9p^2 + \dots) = \frac{(1-p)^3}{(1+p)} \cdot \frac{(1+p)}{(1-p)^3} = 1 \text{ (normalized!)}$

$$b) W_x = x p^{x-1} (1-p)^2, \ln W_x = \ln x + (x-1) \ln p + 2 \ln(1-p)$$

$$\left(\frac{\partial \ln W_x}{\partial x} \right)_p = \frac{1}{x} + \ln p, \quad \frac{1}{x_{\max}} = -\ln p, \quad x_{\max} = \frac{1}{\ln(1/p)}$$

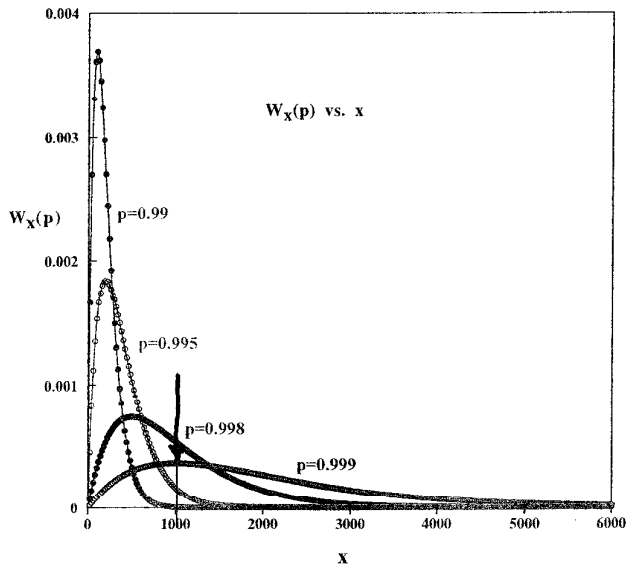
$$x_{\max} = 1000 = \frac{1}{\ln(1/p)}, \quad 10^{-3} = \ln(1/p), \quad \ln p = -10^{-3}, \quad p = e^{-0.001} \\ \approx 1 - 0.001 + \frac{10^{-6}}{2!} - \dots \\ \approx \underline{\underline{0.999}}$$

$$c) \left(\frac{\partial \ln W_x}{\partial p} \right)_x = (x-1) \frac{1}{p} + 2 \frac{(-1)}{(1-p)}, \quad \frac{x-1}{p_{\max}} = \frac{2}{1-p_{\max}}$$

$$(x-1)(1-p_{\max}) = 2 p_{\max}$$

$$x = 500, \quad p_{\max} = \frac{499}{501} = \underline{\underline{0.996}}$$

d)



e)

