Chem 654, Spring 2001

Problem Set #6

Solutions

Problem Set 6 due 04/23/01

1. Tabulated below are the results of stress relaxation measurements made on a sample of bisphenol A poly(carbonate). Construct the master curve and calculate the two WLF constants for a reference temperature of 150.8°C.

T/°C	log t	logE(t)	T/°C	log t	logE(t)
130	2.00	10.25	145.6	2.75	9.03
	2.25	10.25		3.00	8.72
	2.50	10.25		3.25	8.50
	2.75	10.25		3.50	8.25
	3.00	10.22		3.75	8.09
	3.25	10.19		4.00	7.94
	3.50	10.16	146	2.50	8.72
	3.75	10.13		2.75	8.47
	4.00	10.06		3.00	8.25
	4.25	10.03		3.25	8.06
141	2.00	10.19		3.50	7.94
	2.25	10.19		3.75	7.82
	2.50	10.16		4.00	7.67
	2.75	10.13	150.8	2.00	7.97
	3.00	10.09		2.25	7.85
	3.25	10.03		2.50	7.78
	3.50	9.94		2.75	7.72
	3.75	9.82		3.00	7.69
	4.00	9.66		3.25	7.64
	4.25	9.50		3.50	7.59
142	2.00	10.13		3.75	7.55
	2.25	10.09		4.00	7.50
	2.50	10.03		4.25	7.44
	2.75	9.97		4.50	7.34
	3.00	9.88		4.75	7.25
	3.25	9.67	156	2.00	7.59
	3.50	9.63		2.25	7.55
	3.75	9.47		2.50	7.50
	4.00	9.22		2.75	7.41
144.9	2.25	9.66		3.00	7.34
	2.50	9.50		3.25	7.25
	2.75	9.28		3.50	7.19
	3.00	9.03		3.75	7.06
	3.25	8.88		4.00	6.94
	3.50	8.66		4.25	6.78
				4.50	6.59
				4.75	6.38
				5.00	6.13

T/°C	log t	logE(t)	T/°C	log t	logE(t)
159	2.25	7.34	167	2.25	6.72
	2.50	7.25		2.50	6.60
	2.75	7.16		2.75	6.25
	3.00	7.03		3.00	5.97
	3.25	6.94		3.25	5.63
	3.50	6.78		3.50	5.22
	3.75	6.63	171	2.25	6.00
	4.00	6.41		2.50	5.72
	4.25	6.13		2.75	5.41
	4.50	5.85		3.00	5.03
	4.75	5.53		3.25	4.63
161.5	2.25	7.06			
	2.50	6.97			
	2.75	6.85			
	3.00	6.64			
	3.25	6.50			
	3.50	6.31			
	3.75	6.06			
	4.00	5.78			
	4.25	5.47			

2. Creep behavior of a viscoelastic body is commonly modeled by Voigt element. Applied stimulus is a constant stress σ_o over a time interval, $t_1 \le t < t_2$, and $\sigma = 0$ otherwise. Show that the time dependent strain is given by

$$\gamma(t) = \frac{\sigma_o}{G_V}[1 - e^{-(t - t_1)/\tau_V}] \quad \text{for} \quad t_1 \le t < t_2 \tag{1}$$

and

$$\gamma(t) = \frac{\sigma_o}{G_V}[1 - e^{-(t_2 - t_1)/\tau_V}] \cdot e^{-(t - t_2)/\tau_V} \quad (t_2 \le t) \tag{2}$$

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3. A) A Maxwell element under a periodic strain, $\gamma(t) = \gamma_o \cdot e^{i\omega t}$, gives rise to a periodic stress with the same frequency but at a phase lag δ, $\sigma(t) = \sigma_o e^{i(\omega t + \delta)} = \sigma_o^* \cdot e^{i\omega t}$. Prove that the corresponding complex modulus, $\sigma(t)/\gamma(t) \equiv G^*(\omega)$, is given by

$$G^*(\omega) = G_M\left[\frac{\omega^2\tau_M^2}{1+\omega^2\tau_M^2} + i\frac{\omega\tau_M}{1+\omega^2\tau_M^2}\right] = G'(\omega) + iG''(\omega) \qquad (3)$$

B) The parallel case with a Voigt element under a periodic stress, $\sigma(t) = \sigma_o \cdot e^{i\omega t}$, causes a periodic strain, $\gamma(t) = \gamma_o^* \cdot e^{i\omega t}$. Prove that the complex compliance, $\gamma(t)/\sigma(t) \equiv J^*(\omega)$, is expressed as

$$J^*(\omega) = \frac{1}{G_V}\left[\frac{1}{1+\omega^2\tau_V^2} - i\frac{\omega\tau_V}{1+\omega^2\tau_V^2}\right] = J'(\omega) - iJ''(\omega) \qquad (4)$$

C) Plot $G'(\omega)/G_M$ & $G''(\omega)/G_M$ vs. $\log\omega\tau_M$ and $J'(\omega)G_V$ & $J''(\omega)G_V$ vs. $\log\omega\tau_V$.

D) Show that the work done to a Maxwell element for one period is given by

$$W = \oint \sigma(t)d\gamma = \int_0^{2\pi/\omega} \sigma(t)\cdot\dot{\gamma}dt = \pi\gamma_o^2 G'' \qquad (5),$$

and determine the relaxation time τ_M and W if $G_M = 5\cdot10^5$ Pa, $\eta_M = 2\cdot10^3$ Pa·s, $\gamma_o = 0.05$ and f=60Hz.